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A modified NaSch model with density-dependent randomization for traffic flow

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Abstract. Based on the Nagel-Schreckenberg (NaSch) model of traffic flow, a modified cellular automaton (CA) traffic model with the density-dependent randomization (abbreviated as the DDR model) is proposed to simulate traffic flow. The fundamental diagram obtained by simulation shows the ability of this modified NaSch model to capture the essential features of traffic flow, e.g., synchronized flow, metastable state, hysteresis and phase separation at higher densities. Comparisons are made between this DDR model and the NaSch model, also between this DDR model and the VDR model. And the underlying mechanism is analyzed. All these results indicate that the presented model is reasonable and more realistic.

PACS. 45.70.Vn Granular models of complex systems; traffic flow – 05.70.Ln Nonequilibrium and irreversible thermodynamics – 02.60.Cb Numerical simulation; solution of equations – 02.50.Fz Stochastic analysis

Introduction

With the development of modern society, vehicular traffic becomes more and more important in human life. The transportation problems have attracted considerable attention of statistical physicists [1–4] and have been studied with various traffic models, such as cellular automaton (CA) models, car-following models, gas kinetic models, and hydrodynamic models [5–9]. Through theoretical analysis and computer simulation with these models, people have gained deeper insight into the dynamical characteristics of traffic systems and better understanding of the complex phenomena observed in real traffic.

Compared with other traffic models, CA models are conceptually simpler and can be easily implemented on computers for numerical investigations. The related research has been developed very quickly in the last decade after the first realistic CA model was proposed in 1992 by Nagel and Schreckenberg (the NaSch model, for short) [5]. The model deals with single-lane traffic flow of N cars moving in a one-dimensional lattice of L cells under periodic boundary conditions. The number of vehicles is fixed. Each cell may either be empty or be occupied by one car. Each car has an integral velocity between 0 and the speed limit v_{max}. This speed limit may be different depending on the kind of vehicles under consideration. Let v_n and x_n denote the velocity and position of the nth vehicle at the time t respectively. And let d_n be the current empty sites in front of the nth vehicle, $d_n(t) = x_{n+1} - x_n - 1$. Then the state of the system at the time $t+1$ could be obtained from the state at the time t by applying the following set of updating rules:

(1): Acceleration,

$$v_n \to min(v_n + 1, v_{max}).$$

(2): Deterministic deceleration to avoid accidents,

$$v_n \to min(v_n, d_n).$$

(3): Randomization,

$$v_n \to max(v_n - 1, 0) \;\; with\ the\ probability\ p.$$

(4): Update of positions,

$$x_n(t+1) \to x_n + v_n.$$

With the above very simple rules, this model can be used to reproduce the principal phenomena appearing in real traffic, e.g., the phantom traffic jams. However, the maximum flow (i.e., the transit capacity) obtained by numerical simulation with the NaSch model is much lower than 2500 vehicles/(h*lane) which was given by measurements in highway traffic [9]. Moreover, the metastable state with two branches in the fundamental diagram has not been given by the NaSch model. To improve the situation, a number of models have been proposed by introducing the slow-to-start rules or considering the effects of successive vehicles, among which are the VDR

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model [10], the T^2 model [11], the BJH model [12],the FVD model [13] and the XDYD model [14]. Another improved CA model [15] considers the relationship between deceleration probability and the density. The delay probability has been classified into three cases and two different regions were presented, i.e., the coexistence state and the jamming state [16]. Some of these models are able to reproduce the metastable state and exhibit a clear separation of the congestion and free-flow regions in space-time patterns. And the road capacity obtained by numerical simulation approaches to the measured data more closely. Some of these models involve a varying randomization probability, while in the NaSch model, the value of randomization probability is assumed to be constant. So the randomization probability plays an important role in describing traffic dynamics.

However, there have been a few CA models which could be used to simulate the synchronized flow. Recently numerous empirical data of the highway traffic have been obtained, which demonstrated the existence of three distinct dynamic phases: the free traffic flow, the synchronized traffic flow and the traffic jam [17,18]. It has been found empirically that the complexity in traffic flow is linked to diverse space-time transitions between the three basically different kinds of traffic.

Briefly speaking, there has been no traffic model yet which can account for all aspects of vehicular traffic at present. So a wide variety of CA models appeared which described various types of traffic phenomena.

This paper presents a modified NaSch model with the density-dependent randomization for traffic simulation. The fact that the local density of vehicles has influence on drivers' behavior is taken into account. According to the simulation results, we find that the presented model can reproduce the complicated behavior of real traffic, such as the phenomena of synchronized flow, metastable state, hysteresis and phase separation at higher densities. The fundamental diagram obtained by numerical simulation shows that the capacity of the road approaches the empirical data more closely compared with that by the NaSch model.

The rest of the paper is organized as follows. A description of a modified NaSch model with the density-dependent randomization is given in Section 2. In Section 3, simulation results in the forms of fundamental diagrams and space-time patterns are presented and compared to those with the NaSch model and the VDR model. Finally, Section 4 contains concluding remarks and a summary of findings.

Outline of the model

The value of randomization probability p in the NaSch model is assumed to be constant. This means all vehicles on the load have the same braking probability, which does not correspond with real situation. In the VDR model, the delay probabilities are considered as velocity-dependent. Vehicles at the downstream tail of a jam start with reduced probability (slow-to-start) in order to model the restarting process of a vehicle in a more realistic fashion. But few models considered the influence of the density on the randomization probability. In reality, the randomization probability is not only affected by the vehicle velocity, but also affected by the vehicle density. So we propose a modified NaSch model in which a density-dependent randomization probability $p_n = p(\rho_n)$ is introduced to replace the constant p in the original model. This varying probability is assumed to have the following form:

$$p_n = (\rho_n)^r \tag{1}$$

where, p_n is the randomization probability of the nth vehicle; ρ_n is the local vehicle density and expressed as $1/(d_n+1)$, while d_n has been defined above; r is a positive exponent denoting the relationship between ρ_n and p_n. As the right hand of equation (1) is a power function and the value of $1/(d_n+1)$ is between 0 and 1, then the value of p_n is also between 0 and 1. And this coincides with the property of the randomization probability. Any number is suitable for r in principle provided that it is greater than zero. But here the values of r are set to be greater than 1 on account of the fact that the curve of the power function appears a concave one as $r \geq 1$. This means that when the local density is small, i.e., the distances between successive vehicles are large, the corresponding randomization is small according to the function, then p_n increases slowly with the increase of ρ_n in the middle region of $[0,1]$ and increases drastically when ρ_n approaches 1. And this characteristic reflects the real traffic behavior. Here r is set to be from 1 to 10 in order to make the probability not too large or too small.

The numerical simulation was performed according to the above updating rules which are the same as those of the NaSch model besides the modification of the randomization probability. A one-dimension lattice of L cells was examined with periodic boundary conditions, and only one kind of vehicles moving along one direction was considered. Each cell was set to be 7.5 m long and either empty or occupied by just one vehicle. The value of vehicle velocity was between 0 and $v_{max}(=5)$, where v_{max} was the same for all vehicles. A system of length $L = 1000$ was studied, which corresponded to the length of actual road around 7.5 km. One time step Δt corresponded to $1s$, which was of the order of the reaction time for humans. Then, the maximum velocity $v_{max} = 5$ corresponded to 135 km/h in real traffic.

The computational formulas used in numerical simulation are given as follows:

$$J = \rho \cdot v \tag{2}$$

where J is mean flow, ρ is global vehicle density, and v is mean velocity. Then ρ and v can be expressed respectively as

$$\rho = N/L, \tag{3}$$

$$v = \frac{1}{T}\sum_{t=t_0}^{T+t_0-1}\frac{1}{N}\sum_{n=1}^{N} v_n(t), \tag{4}$$

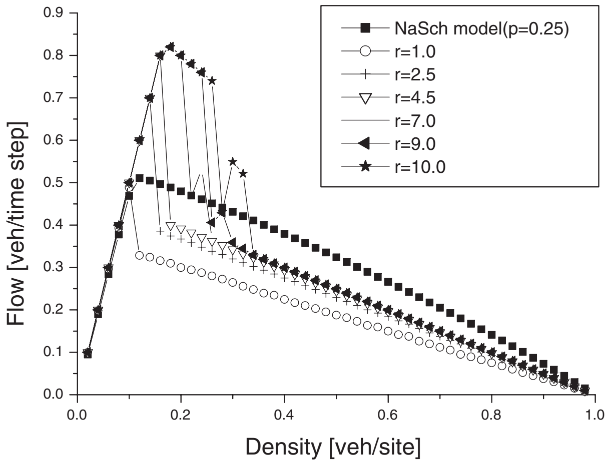

Fig. 1. Fundamental diagrams obtained from the DDR model ($v_{max} = 5, L = 1 \times 10^3$) with different values of r and from the $NaSch$ model ($v_{max} = 5,\ p = 0.25; L = 1 \times 10^3$) under the same initial condition.

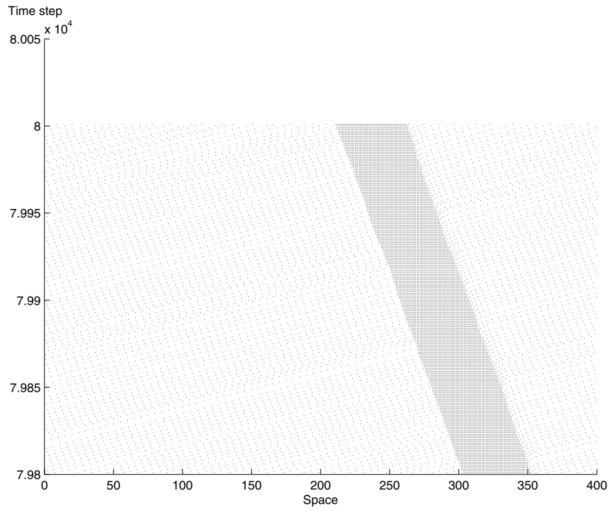

Fig. 2. The space-time pattern obtained from the DDR model with $v_{max} = 5,\ r = 7, L = 1 \times 10^3, \rho = 0.2$. The plot apparently shows the spontaneous formation of jams. (The horizontal direction in space is 400 cells and time increases in the vertical upward direction from 7.98×10^4 to 8×10^4 after removing the transient effects.)

where N is the total vehicles distributed on the selected road, $v_n(t)$ is the velocity of the nth vehicle at time t, and T is selected time interval. In the numerical simulation, the first 1×10^4 time-steps of each run were put away in order to remove the transient effects, and then the data were recorded in successive 1×10^4 time-steps. The mean velocity was obtained by averaging over 30 runs.

Simulation results

First the fundamental diagrams of our proposed model are presented in Figure 1 with different values of r which is defined above. It is found that for $1 \leq r \leq 4.5$, the curves show the similar reverse-lambda shape as that in the VDR model and the maximum flow increases with the increase of r. This is because that the probability decreases with the increase of r. For $r \geq 7$, the curves are different in the following three regions and the maximum flow remains constant. In the first region or low-density region, the flow increases linearly with the increase of density. In the second region the forms of the diagrams are significantly different from that with $1 \leq r \leq 4.5$ and the non-trivial phenomenon is that the mean total flow J shows only a slight decrease with the increasing of the density. Then the flow drops abruptly and fluctuates around some smaller values. In the third region, i.e., the high density region, the traffic flow decreases linearly with the increase of vehicle density.

Actually, as $r \geq 7$, the randomization probability approaches the value of 0 and 1 according to the definition of the probability. When the global density is small, the local density is also small and probability is almost 0. Then vehicles travel at desirable speed in free flow phase. This corresponds to the first region in Figure 1.

With increasing global density, the local density increases somewhere and the randomization probability may be close to 0 or 1. Then the phase separation phenomenon is presented in Figure 2, in which the corresponding density is 0.2 and the road length is set to be 400 in order to make the diagram clearer. The gray region corresponds to free flow and the dark region corresponds to the standing vehicles that cluster to form the jam. Some vehicles cannot keep their desired velocity due to various reasons including the large local density and frequently decelerate at random. The fluctuations of velocity will cause some vehicles to stop, thereby forming a jam. The free flows are evidently separated by jams. This phenomenon is called separation of phases, which usually makes the vehicles slow down and the capacity drop. This phenomenon was also appeared in the VDR model.

But sometimes the capacity does not drop much due to the phase separation, just as that shown in Figure 1. In order to have some further insight into the dynamics, a fragment of a time pattern of the average speed of vehicles at the chosen section is presented in Figure 3, in which the global density is 0.2 and 0.32. It is found that vehicles move almost at the same velocity within the time period, although the velocity is smaller than that in the free flow phase. This is the phenomenon of synchronized flow. Meanwhile the average speed as $\rho = 0.2$ is larger than that as $\rho = 0.32$. And this is in good agreement with the real traffic. Then the existence of the second region in Figure 1 should be ascribed to the presence of this synchronized flow phase. The slight decrease of flow with the increasing of the density is due to the jamming cluster. It should be pointed out that the reproduced synchronized flow with the DDR model is the result of combined action of phase separation and density-dependent randomization. And this is the main reason why there exists phenomenon of phase separation but can not reproduce the synchronized flow with the VDR model.

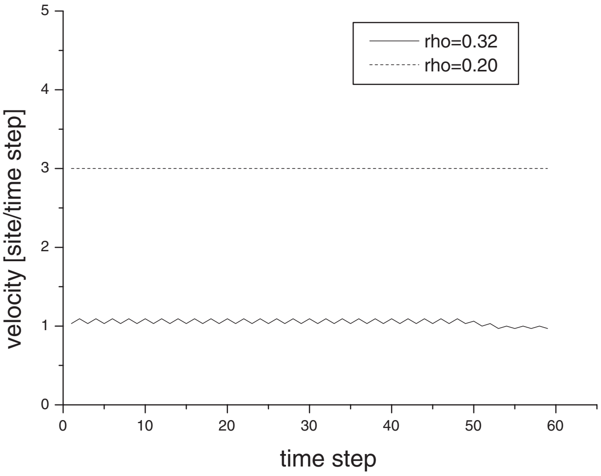

Fig. 3. A fragment of a time pattern of the average speed of vehicles at the chosen section obtained by the DDR model with $v_{max} = 5,\ r = 9; L = 400; \rho = 0.2$ and $\rho = 0.32$.

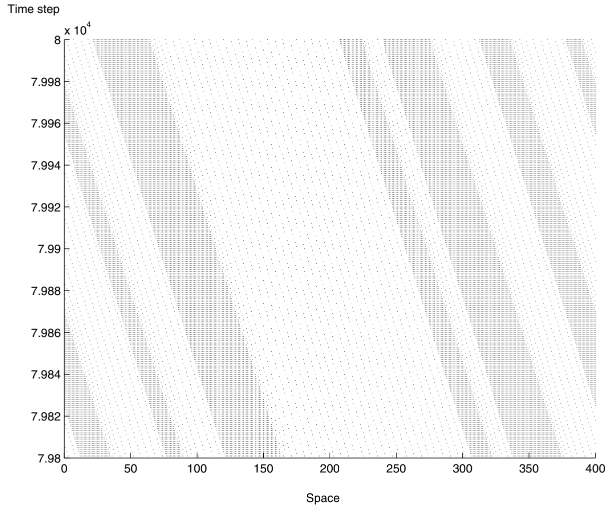

Fig. 4. The space-time pattern obtained from the DDR model with $v_{max} = 5, r = 9, L = 400, \rho = 0.4$. The plot apparently shows the successive forming jams. The horizontal direction in space is 400 cells and time increases in the vertical upward direction from 7.98×10^4 to 8×10^4 after removing the transient effects.

It has been found when traffic flow transits from the free flow phase to the synchronized flow phase(the $F \rightarrow S$ transition, for short) with average speed decreasing, the flow rate in the emerged synchronized flow can remain of the same order of magnitude as in the initial free flow [20]. And this phenomenon coincide with the simulation result in Figure 1.

The space-time diagram with $r = 9$ and $\rho = 0.4$ is presented in Figure 4. It is found that there are a series of jam forming on the road. That is to say, more vehicles will stop forming successive jams and the synchronized flow transits to a stop-and-go jam. With further increase of density, the successive small jams transform into a wide moving jam. The reason for this transition is the increasing density and reduction of the outflow from a jam. As the outflow from a jam is not very large, or the outflow is smaller than the global flow, the small-width jams can not dissolve and merge into a wide jam. Then the flow presents a tendency of linear decrease with the increase of vehicle density due to the wide moving jam, as the third region shown in Figure 1.

It is important to note that with the increase of the value of r, the range of density corresponding to the synchronized flow phase becomes wider. This phenomenon is linked to the fact that randomization probability decreases with r more drastically with increasing density. That is to say, the larger the value of r, the wider range of density with small randomization is. Then the traffic will be in the synchronized flow for wider range of density region subsequently. Furthermore the metastable state exists in the fundamental diagram of the DDR model that will be discussed later.

At low densities, the slopes of the fundamental diagrams for all values of r are similar; indicating that vehicles travel at near maximum speed in free flow phase and the flow is independent of r. While in the jamming phase the slope is also the same except the case of $r = 1.0$, indicating a similar traffic behavior in this region and the value of $r = 1.0$ is not suitable because of the lower value of the maximum flow. By taking into account the appearance of the synchronized flow phase, the numbers of r between 7 and 10 are recommended.

From the above analysis, we can find that three regions in Figure 1 correspond to the three distinct dynamic phases that are observed in real traffic: the free flow phase, the synchronized flow phase and the jamming phase.

For comparison the fundamental diagram obtained from the NaSch model with $p = 0.25$ is also presented in Figure 1, which shows a direct transition from a free flow regime to a jammed regime with increase of vehicle density.

Besides the phenomena of synchronized flow and metastable state, other two qualitative differences between the DDR model and the NaSch model are found. First our model leads to a higher value of maximum flow than that obtained from the NaSch model by 40%, which is close to the observed data (2500 vehicles/h*lane) [9]. Second, at small density or free flow phase, the relationships between density and flow in the two models are quite the same. While at higher density or in the jamming phase, flow in the DDR model is lower than that in the NaSch model corresponding to the same value of density. Furthermore in the congestion regime the flow decreases with density linearly in the DDR model, while in the NaSch model the flow-density curve is always convex in the large-density region.

In the DDR model, when the condition $d_n \geq 1$ is satisfied, the randomization probability of vehicle is less than 0.25 as $r > 2$ according to equation (1). And in the low-density region corresponding to the maximum flow, the randomization probability in the DDR model is less than that in the NaSch model (=0.25). Then the reduction of braking probability leads to the higher value of maximum flow.

The reason for the second phenomenon is that at low densities ($\rho \ll 1$) there are no slowly moving vehicles in both models since the interactions between vehicles are extremely rare. Then the randomization probability has little influence on the total flow and the fundamental diagrams of the two models show the same property. On the other hand, at larger density ($1-\rho \ll 1$), the flow is given by $J(\rho) \approx (1-p_n(\rho_n))(1-\rho)$ in the DDR model and $J(\rho) \approx (1-p)(1-\rho)$ in the $NaSch$ model. Because for densities close to $\rho = 1$, vehicles can only have velocities of $v_j = 0$ or $v_j = 1$. Thus $p_n(\rho_n) > p(=0.25)$ when $d_n < 1$ according to equation (1), and the flow in the DDR model is smaller than that in the $NaSch$ model. It can be interpreted in another way that when the density of vehicles is getting larger, vehicles will likely decelerate. This will make the local density even larger and the traffic circumstances even worse. At that time the randomization probability is certainly larger than that in the light traffic condition. So the concept of density-dependent randomization is in a more realistic fashion than the constant randomization. Besides there is some observational evidence that in certain situations the shape of fundamental diagrams differs from the convex form [10], such as those in the slow-to-start models [10–12].

While in the VDR model, the randomization probabilities are considered as velocity-dependent. The modification of randomization probability is in the following form:

$$p(v) = p_0, \quad v = 0 \tag{5}$$

$$p(v) = p, \quad v > 0. \tag{6}$$

Then the VDR model reproduced not only the phenomena of metastability and hysteresis, but also phase separated states at higher densities which are also presented in the DDR model.

In order to confirm the existence of metastable state, two basic strategies can be adopted. First, the density of vehicles is changed adiabatically by adding or removing vehicles from the stationary state at a certain density. The second way does not require changing the density. Instead one starts from two different initial conditions, a completely jammed state (megajam) and a homogeneous state. The megajam state consists of one large compact cluster of standing vehicles. While in the homogeneous state, vehicles are distributed equidistantly.

Then we compare the dynamics for the DDR model and the VDR model. Obviously the synchronized flow phase exhibited in the DDR model is the most distinct feature compared with the VDR model. Except for this issue the two models have some similar properties. For comparison, Figure 5 is the simulation result with our DDR model for the case of $v_{max} = 5, r = 9$ using the second strategy described above. The same has been done in Figure 6 which is the fundamental diagram of the VDR model with $v_{max} = 5, p_0 = 0.75$ and $p = 1/64$. The system size used for simulations was $L = 1000$. It is shown that discontinuous reduction of traffic flow at the critical density

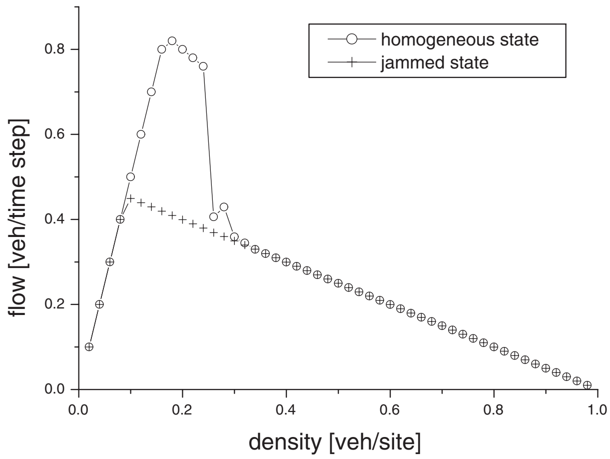

Fig. 5. The fundamental diagrams in the DDR model ($v_{max} = 5,\ r = 9; L = 1 \times 10^3$) obtained by using two different initial conditions, namely, a completely jammed state and a homogeneous one. The metastable state occurs.

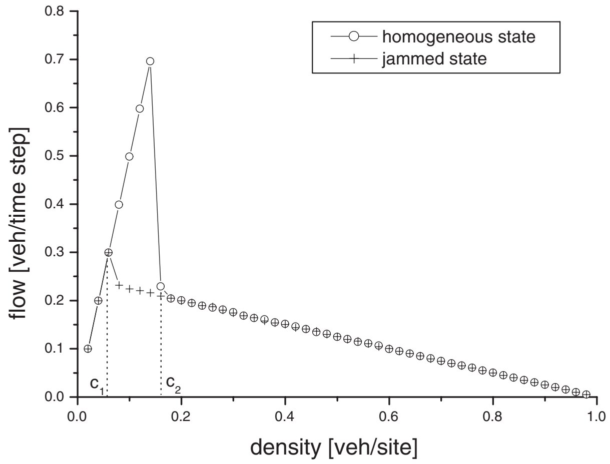

Fig. 6. The fundamental diagrams of the VDR model ($v_{max} =\ 5,\ p_0 = 0.75, p = 1/64, L = 1 \times 10^3$) obtained by using two different initial conditions, namely, a completely jammed state and a homogeneous one. (the left dotted line corresponding to ρ_1, and the right dotted line corresponding to ρ_2).

ρ_c is observed in VDR model, where $\rho_c = 1/(1+v_{max})$. Below the critical density, traffic flow reaches a maximum. In certain density region a reverse-lambda shape in the fundamental diagram can be observed which is absent in the NaSch model. The left leg of the reverse-lambda (with higher flow) which corresponds to a homogeneous free-flow state; there are almost no interactions between the vehicles. On the other hand, the right leg of the reverse-lambda, traffic flow begins to depart from a linear increase at the density c_1 and even decreases with a further increase of vehicle density, where c_1 represents the transition density from the freely moving phase to the jamming phase in the case of the initial inhomogeneous condition. This is called metastable state and the hysteresis phenomena are always related to the existence of metastable states in certain density region. This means that traffic behaves differently in different density regimes and the uncongested and congested regimes are separated by gaps or discontinuities which have been observed from the empirical works [19–21]. Such situation is also described as a two-capacity phenomenon. One capacity corresponds to the tip of the left leg of the reverse-lambda, and the other belongs to the tip of the right leg of the reverse-lambda with a capacity drop from the former tip. And this capacity drop is believed to be caused by the formation of queues on roads.

The metastable state also appears in the fundamental diagram obtained from the DDR model (see Fig. 5). Under a same global density, the local density for a certain vehicle is usually different from the process of increasing density to the process of decreasing density. And the randomization is different due to the density-dependent probability. Then flow can take one of the two values over a certain interval of density, and metastability exhibits. That is to say, the value of flow is dependent on the initial state. While in the NaSch model, the metastability does not appear due to the constant probability.

From Figure 5 it is found that the shape of the diagram is a bit different from a reverse-lambda, i.e., there is an extension to the left leg with a slight decrease flow. And this extension part corresponds to the synchronized flow. Along the left leg of the reverse-lambda, traffic flow gradually increases with density. At a critical density, the flow reaches its maximum corresponding to the tip of the reverse-lambda. Then the synchronized flow appears with the further increase of density in which the flow maintains rather high values. When the density of traffic continues to increase, the discontinuous reduction of traffic flow or metastable state occurs behind the synchronized flow regime; and this is the unique characteristic that is different from the other CA models including the VDR model. It is found that the flow decreases along the right leg of the reverse-lambda. The transition from the extension part of the left leg to the right leg signifies the occurrence of a heavy downstream queue. When the queue starts to discharge, the data points moved up from the bottom of the right leg and finally transits back to the left leg but can not transit back to the extension part of the left leg (synchronized flow). That is to say, the maximum flow value and the synchronized flow phase prior to the influence of the large queue can not be recovered. This demonstrates the two-capacity phenomenon as observed in real traffic flow data.

Conclusions

In this paper, a modified NaSch model with density-dependent randomization has been proposed to simulate microscopic traffic flow. The simulation results indicate that this model can reproduce the complicated traffic behavior of real traffic, such as the phenomena of synchronized flow, metastable state, hysteresis and phase separated states at high densities. Comparisons have been made between this DDR model and the VDR model, also between the DDR model and the NaSch model. And the underlying mechanism has been analyzed. Furthermore the fundamental diagram obtained by numerical simulation shows that the capacity of the road approaches the empirical data more closely compared with that from the NaSch model. We should stress that our model is rather powerful in dealing with realistic traffic flow phenomena, because it takes into account the local density in the determination of the randomization probability. And this coincides with the real behavior of drivers. Especially the reproduced synchronized flow is a biggest challenge for traffic flow models which is absent in most CA models. The results show that the presented model is more reasonable and realistic.

Although in this paper the model is simulated in a simple one-dimension topology with one type of vehicle, it is possible to apply it to complex highway topologies including multi-class traffic with different types of vehicles. Moreover it is possible to apply it to stabilize the homogeneous branch of the fundamental diagram to maximize the throughput.

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